Code . AV – 5017

B. Tech Ist Year 2015–16

Model Answer (Old Course)

Note There may be alternate solution to any problem.

Section A

1 (i) Lagrange's Mean Value Theorem

If a function $f(x)$ is continuous in closed interval $[a,b]$ and $f'(x)$ exists in (a,b), Then there exists one value c of x in (a,b) such that

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

(ii) If $y = e^{ax}\sin(bx+c)$, then

$y_n = r^n e^{ax}\sin(bx+c+n\phi)$ where $r^2 = (a^2+b^2)$

and $\phi = \tan^{-1}\frac{b}{a}$

(iii) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots$

(iv) Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right) = \lim_{x\to 0}\frac{x-\sin x}{x\sin x} = 0$

(v) If $I_n = \int \tan^n x\,dx$, Then

$$I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$$

(vi) If $I_n = \int \operatorname{Cosec}^n x\,dx$, then

$$I_n = \frac{\cot x \operatorname{Cosec}^{n-2}x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

(vii) To find asymptotes $\parallel^{el}$ to x-axis, equate to zero the coefficients of highest powers of x in the equation, provided this is not constant.

... of the curve $x^2y^2 = a^2(x^2+y^2)$ ... $x=a$ line

(ix) $dy = \left(e^{3x-2y} + x^2 e^{-2y}\right) dx$ (Variables Seperable)

$e^{2y}\, dy = \left(e^{3x} + x^2\right) dx$

$\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + \text{const.}$

(x) Bernoulli's Eqn

The equation

$\frac{dy}{dx} + Py = Qy^n$

where P & Q are functions of x only.

Section B

2 (a) If $y = x \log \frac{x-1}{x+1}$, then

Exp. 4.5 / Page 137 B.S. Grewal Higher Engg. Maths 43rd Edition

$y_1 = \log \frac{x-1}{x+1} + x\left[\frac{1}{x-1} - \frac{1}{x+1}\right]$

$\Rightarrow y_1 = \log(x-1) - \log(x+1) + \frac{1}{x-1} + \frac{1}{x+1}$

$\Rightarrow y_n = (-1)^{n-2} \lfloor n-2 \left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

(b) $y = \tan^{-1}\frac{2x}{1-x^2}$, Put $x = \tan\theta$

Q. 11 / Page 138 B.S. Grewal 43rd Edition

$\Rightarrow y = 2\theta = 2\tan^{-1}x$

$\Rightarrow y_n = 2(-1)^{n-1} \lfloor n-1 \sin n\alpha \sin^n\alpha$

where $\alpha = \cot^{-1}x$

3 (a) Example 4 / Page 159, B.S. Grewal

(b) $f(x) = \sin x(1 + \cos x)$

Exp 4.57 / Page 178 B.S. Grewal 43rd Edition

$f'(x) = (1 + \cos x)(2\cos x - 1)$

... or $\frac{1}{2}$

-3-

(a) $y = \lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$ $\frac{Q(ii)}{\text{Page 156}}$ B. S. Grewal 43rd Edition

$\log y = \lim_{x \to 0} \frac{\log(a^x + b^x + c^x) - \log 3}{x}$

$= \lim_{x \to 0} \frac{(a^x + b^x + c^x)^{-1} (a^x \log a + b^x \log b + c^x \log c)}{}$

$= (1+1+1)^{-1} (\log a + \log b + \log c)$

$= \log (abc)^{\frac{1}{3}}$

$\therefore y = (abc)^{\frac{1}{3}}$

(b) $\frac{\text{Exp. 4.66 (ii)}}{\text{Page 184}}$ B. S. Grewal 43rd Edition

$m = 1, -\frac{1}{2}, -\frac{1}{2}$

$c = 0$ when $m = 1$

$c = \pm\frac{1}{2}$ when $x = -\frac{1}{2}$

The asymptotes are

$y = x$, $y = -\frac{1}{2}x + \frac{1}{2}$, $y = -\frac{1}{2}x - \frac{1}{2}$

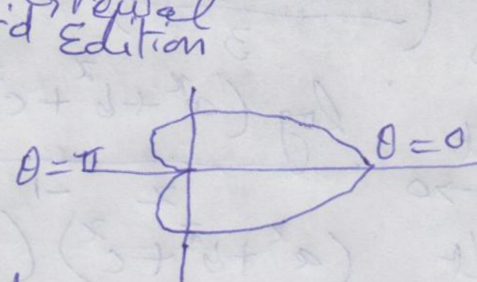
5 (a) $\frac{\text{Exp. 4.48}}{\text{Page 169}}$ B. S. Grewal 43rd Edition

(b) $\frac{\text{Exp. 5.6}}{\text{Page 201}}$ B. S. Grewal 43rd Edition Part I (Solution)

6 (a) $\frac{\text{Exp. 6.10}}{\text{Page 248}}$ B. S. Grewal 43rd Edition

IFT $= \int^{\pi/2} \cos^m x \cos nx \, dx$

(b) Exp. 6.34 / Page 264 B.S. Grewal 43rd Edition

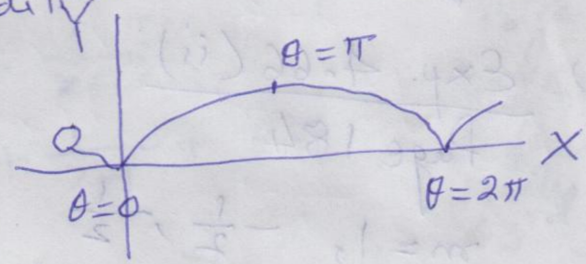


Length

$$= 2\int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= 8a$$

7 (a) Exp. 6.23 / Page 259 B.S. Grewal 43rd Edition



Required Area

$$= \int_{x=0}^{x=2\pi a} y\, dx$$

$$= 8a^2 \int_{\theta=0}^{\theta=\pi} \sin^4 \frac{\theta}{2}\, d\theta$$

$$= 3\pi a^2$$

(b) Exp. 7.43 / Page 305 Part (i) B.S. Grewal, 43rd Edition

$$\left\{ c^x = e^{\log c^x} = e^{x \log c} \right.$$

$$\int_0^{\infty} \frac{x^c}{c^x}\, dx = \int_0^{\infty} \frac{x^c}{e^{x\log c}}\, dx$$

$$= \int_0^{\infty} e^{-x\log c}\, x^c\, dx$$

Put $x \log c = t$

$\therefore dx \log c = dt$

$\Rightarrow dx = \frac{dt}{\log c}$

$$= \frac{1}{(\log c)^{c+1}} \int_0^{\infty} t^c e^{-t}\, dt$$

(a) $\underline{\text{Exp. } 11.23}$ B.S. Grewal 43 Edition

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dividing $\cos^2 y$ throughout, we get

$$\sec^2 y \frac{dy}{dx} + 2x \frac{\sin y \cos y}{\cos^2 y} = x^3$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$$

Solution is $\tan y = \frac{1}{2}(x^2 - 1) + c e^{-x^2}$

(b) $\underline{\text{Exp. } 11.11}$ B.S. Grewal 43^{rd} Edition

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$$\frac{dy}{dx} = \left(\frac{y}{x} \sec^2 \frac{y}{x} - \tan \frac{y}{x}\right) \cos^2 \frac{y}{x}$$

Put $y = vx$

$$\Rightarrow x \frac{dv}{dx} = \cancel{v} - \tan^2 v \sec v - \cancel{v}$$

Variables seperable,

$$\frac{\sec^2 v}{\tan v} dv = \frac{dx}{x}$$

$$\Rightarrow x \tan \frac{y}{x} = C$$

9(a) Taking $\frac{d}{dx} = D$, we write

$$(D-7)x + y = 0 \quad \text{--- (1)}$$

$$-2x + (D-5)y = 0 \quad \text{--- (2)}$$

Eliminate x as if D were an algebraic multiplier.

Multiplying (1) by -2 and (2) by $(D-7)$, we get

$$-2(D-7)x - 2y = 0$$

$$\ldots + (D-7)(D-5)y = 0$$ on subtracting

(b) $(D^3 - D^2 - 6D)y = x^2 + 1$

A.E is $m^3 - m^2 - 6m = 0$

$m(m^2 - m - 6) = 0$

$m(m-3)(m+2) = 0$

Then either $m = 0$ or $m = 3$ or $m = -2$

C.F. $y = c_1 + c_2 e^{3x} + c_3 e^{-2x}$

$$P.I. = \frac{1}{-6D - D^2 + D^3}(1+x^2)$$

$$= \frac{1}{-6D}\left\{1 + \frac{1}{6}D - \frac{1}{6}D^2\right\}^{-1}(1+x^2)$$

$$= -\frac{1}{6D}\left\{1 - \frac{1}{6}D + \frac{7}{36}D^2 + \cdots\right\}(1+x^2)$$

$$= -\frac{1}{6D}\left\{(1+x^2) - \frac{1}{6}D(1+x^2) + \frac{7}{36}D^2(1+x^2) \cdots\right\}$$

$$= -\frac{1}{6D}\left\{x^2 - \frac{1}{3}x + \frac{25}{18}\right\}$$

$$P.I = -\frac{1}{6}\left\{\frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^2}{2} + \frac{25}{18}x\right\} \quad \text{as } \frac{1}{D}\cdot x = \int x\,dx$$

Complete solution is

$$y = c_1 + c_2 e^{3x} + c_3 e^{-2x} - \frac{1}{6}\left\{\frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^2}{2} + \frac{25}{18}x\right\}$$

10 (a) Article 18.7 B.S. Grewal 43rd Edition

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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{——— (1)}$$

Let $u = X(x)Y(y)$ be the solution of (1)

we get $\frac{d^2X}{dx^2} - kX = 0$ & $\frac{d^2Y}{dy^2} + kY = 0$

The possible solutions are

$u = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

$u = (c_5 \cos px + c_6 \sin px)(c_7 e^{py} + c_8 e^{-py})$

$u = (c_9 x + c_{10})(c_{11} y + c_{12})$

... be obtained

Let $U(x,y) = X(x)\,Y(y)$ be the solution

$$\Rightarrow \frac{\partial U}{\partial x} = \frac{dX}{dx} Y$$

$$\Rightarrow \frac{\partial U}{\partial y} = \frac{dY}{dy} X$$

Putting in equation, we get

$$\frac{dX}{dx} Y = 5 \frac{dY}{dy} X$$

$$\Rightarrow \frac{1}{X}\frac{dX}{dx} = 5 \frac{dY}{dy}\cdot\frac{1}{Y} = k \text{ (say)}$$

which implies

$$\frac{1}{X}\frac{dX}{dx} = k \Rightarrow \frac{dX}{dx} = kX \quad \text{— (1)}$$

$$\text{and } \frac{dY}{dy} = \frac{k}{5} Y \quad \text{— (2)}$$

from (1), we get

$$\frac{dX}{X} = k\,dx \Rightarrow \log X = kx + c_1$$

$$\Rightarrow X = e^{kx + c_1} = A e^{kx} \quad \text{— (3)}$$

Again $\frac{dY}{Y} = \left(\frac{k}{5}\right) dy \Rightarrow \log Y = \left(\frac{k}{5}\right) y + c_2$

$$\Rightarrow Y = e^{\left(\frac{k}{5}\right) y + c_2} = B e^{\frac{k}{5} y} \quad \text{— (4)}$$

Thus solution from (3) & (4) is

$$U(x,y) = \lambda e^{kx + \frac{k}{5} y} \quad \text{— (5)} \qquad \text{where } \lambda = AB$$

Boundary condition is

$$U(0,y) = 10 e^{-7y} \quad \text{— (6)}$$

from (5) & (6), we get

$$10 e^{-7y} = \lambda e^{0.k} \cdot e^{\frac{k}{5} y}$$

11 (a) Subsidiary Eq. are

Exp 17.1) Page 583 S.K.Gr 43rd Edition

$$\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy} \quad \text{— (A)}$$

$$\Rightarrow \frac{dx-dy}{x^2-y^2-(y-x)z} = \frac{dy-dz}{y^2-z^2-x(z-y)}$$

$$\Rightarrow \frac{d(x-y)}{(x-y)(x+y+z)} = \frac{d(y-z)}{(y-z)(x+y+z)}$$

$$\Rightarrow \log(x-y) = \log(y-z) + \log c$$

$$\Rightarrow \frac{x-y}{y-z} = c \quad \text{—— (1)}$$

Each of subsidiary eq. (A)

$$= \frac{x\,dx+y\,dy+z\,dz}{x^3+y^3+z^3-3xyz} = \frac{x\,dx+y\,dy+z\,dz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)} \quad \text{— (C)}$$

Also $\dfrac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}$ — (D)

Thus equating (C) & (D), we get

$$\frac{x\,dx+y\,dy+z\,dz}{x+y+z} = dx+dy+dz$$

$$\Rightarrow \int (x\,dx+y\,dy+z\,dz) = \int (x+y+z)\,d(x+y+z) + c'$$

$$\Rightarrow x^2+y^2+z^2 = (x+y+z)^2 + 2c'$$

$$\Rightarrow xy+yz+zx = c'$$

Thus $\phi\left(\dfrac{x-y}{y-z}\right) = f(xy+yz+zx)$

(b) The diff. Eq. is

$$(D+2D')(D+D')z = x+y$$

Complete Solution is

$$z = \phi_1(y-2x) + \phi_2(y-x) + \frac{x^2 y}{2} - \frac{x^3}{3}$$

(c) A.E. is

$$\frac{dx}{yz} = \frac{dy}{-xz} = \frac{dz}{xy}$$

Thus $x\,dx + y\,dy = 0$

On integrating, we get

$$x^2 + y^2 = a$$

Taking first and last members, we get

$$x\,dx + z\,dz = 0$$

On integrating we get

$$x^2 + z^2 = b.$$

Thus required solution is

$$\phi(x^2+y^2, x^2+z^2) = 0$$

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